Chapter 4

Frequency Hopped Spread Spectrum

In the previous chapter we introduced the concept of direct sequence spread spectrum. While this is the most common form of spread spectrum in commercial systems, it is by no means the only important technique. In this chapter we describe a second common form, which is more common in military systems namely frequency hopped spread spectrum (sometimes also called frequency hopping spread spectrum). Additionally, we will also briefly describe other techniques including chirp modulation and time-hopping.

4.1 Definition/Description

The fundamental goal of spread spectrum systems is to increase the dimensionality of the signal. By increasing the dimensionality, we make eavesdropping and/or jamming more difficult since there are more dimensions of the signal to consider. In commercial applications, the increased dimensionality provides robustness in the presence of other systems and less interference caused to those same systems. It also provides more robustness in fading channels. The main method of increasing the dimensionality of the signal is to increase the signal’s spectral occupancy. Last chapter we discussed in detail one method of accomplishing this, direct sequence spread spectrum. In DS/SS the bandwidth is increased by directly multiplying the data signal by a high-rate pseudorandom spreading sequence. In the frequency domain this results in the convolution of the data spectrum with the spectrum of the spreading signal. The resulting spectrum is significantly more broad than the original data signal spectrum. A second method of accomplishing this bandwidth expansion is through frequency hopping. In frequency hopped spread spectrum (FH/SS) the carrier frequency of the data modulated sinusoidal carrier is periodically changed over some predetermined bandwidth in a pseudorandom manner. By "hopping" the center frequency to one of $N$ (usually but not necessarily) contiguous but non-
overlapping frequency bands, the overall spectrum occupancy is increased by the factor \( N \). It is important that the hopping is typically done in a pseudo-random manner. In military applications this makes interception and jamming more difficult. In commercial applications, it reduces the impact of a particular co-channel interferer as well as the impact of the frequency hopping signal to another system since it will only be present in a particular band on average \( \frac{1}{N} \) of the time.

The hopping signal can be represented as

\[
    h(t) = \sum_{i=-\infty}^{\infty} p(t - iT_c)\cos(2\pi f_i t + \phi_i)
\]

where \( p(t) \) is the pulse shape used for the hopping waveform (typically assumed to be a square pulse), \( f_i \in \{f_1, f_2, \ldots, f_N\} \) are the \( N \) hop frequencies, \( T_c \) is the hop period (sometimes also called the chip period), and \( \phi_i \) are the phases of each oscillator. Note that unlike in DS/SS systems, the chip period (or hop period) does not impact the bandwidth expansion. Bandwidth expansion is completely determined by the number of hop frequencies. The resulting frequency hopped transmit signal is then

\[
    s(t) = [s_d(t)h(t)]_{BP F} = \left[ s_d(t) \sum_{i=-\infty}^{\infty} p(t - nT_c)\cos(2\pi f_i t + \phi_i) \right]_{BP F}
\]

where \( s_d(t) \) is the bandpass data signal which depends on the modulation scheme employed and the bandpass filter (applied to the quantity within \([ \cdot ]_{BP F} \)) is designed to transmit the sum frequencies only. The concept of frequency hopping is illustrated in Figure 4.1. As time advances the signal occupies a separate frequency band as determined by the pseudorandom hopping sequence. On average the power spectral density is spread over the entire band as shown. Provided that each frequency band is used \( \frac{1}{N} \) of the time, the spectrum will be similar to that seen in DS/SS systems when averaged over a sufficiently long time period.

The transmitter and receiver for a typical implementation are shown in Figures 4.2 and 4.3 respectively. As shown in the figures, any modulation scheme (with either coherent or non-coherent demodulation) can theoretically be used. As in DS/SS the frequency hopping is ideally transparent to the data demodulator. The data modulated carrier is hopped to one of \( N \) carrier frequencies every “chip” period \( T_c \) which may be longer or shorter than the data symbol period \( T_s \). At the receiver the same pseudorandom hopping pattern is generated such that the received signal is ideally mixed back down to the original carrier frequency. Data demodulation is then accomplished as in standard digital communications. Note that the bandwidth expansion factor is equal to \( N \) the number of hop frequencies. Unlike in DS/SS, the bandwidth expansion is not dependent on the chip period \( T_c \). In fact, as mentioned, the chip period can
be greater than the symbol period. In other words, the hopping may be slower than the symbol rate. We will discuss the consequences of this relationship later.

### 4.1.1 Modulation

Although, any modulation format can be used with FH/SS, coherent demodulation techniques require that the receiver maintain frequency coherence each hop. This can be difficult to maintain and thus non-coherent demodulation techniques are more commonly used with FH/SS. Specifically, M-FSK is commonly used in conjunction with FH/SS. When FSK is combined with frequency hopping, the system is a frequency domain analog to DS/SS. In DS/SS systems both data and the spreading code are used to modulate the phase of the carrier. In FH/SS with FSK, both data and the spreading code are used to modulate the frequency of the carrier. This can be seen in Figure 4.4 where $T_c = 4T_s$. M-FSK modulation is used and $N = 6$. The hopping sequence determines which of $N$ bands is used, while the data determines which of the four frequencies within the band is transmitted.
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4.1.2 Slow versus Fast Hopping

As mentioned earlier, the hop period (also called the chip period $T_c$) may be greater or less than the symbol duration. The bandwidth expansion factor (and consequently the processing gain) is related only to the number of hop frequencies $N$, not the hop period. Thus, we are free to choose the hop period based on other considerations. Specifically, the hop frequency should be chosen based on implementation and performance considerations. First let us consider the case where $T_c > T_s$, which is called slow hopping. Additionally, let us assume that FSK modulation is used. Figure 4.4 plots the frequency occupancy versus time considering both the data modulation and frequency hopping. In this example $T_c = 4T_s$, or the frequency is hopped every four symbols, $N = 6$ and $M = 4$ ($T_b = T_s/2$). Further, in the figure we have defined $B$ as the bandwidth of the MFSK signal and $W$ as the spread bandwidth. As can be seen, every $T_c$ seconds, the frequency is changed to one of 4 symbols based on the data. Additionally, every $T_b$ seconds, the center frequency of these symbols is changed based on the frequency hopping pattern. At the receiver the pseudorandom hopping is removed, leaving only the data modulation as shown in Figure 4.5.

Figure 4.4: Example of a Time-Frequency Plot for Slow Hopping ($T_c=$Hop period, $T_s =$ symbol period, $T_b =$ bit period, $W =$ spread bandwidth, $B =$ symbol bandwidth)
In contrast to slow hopping, with fast frequency hopping \( T_c < T_s \). That is, frequency hopping occurs faster than the modulation. This is depicted in Figure 4.6 where \( T_c = \frac{T_s}{2} \), \( N=6 \), and \( M=4 \). In this case coherent modulation is extremely difficult since it would require extremely fast carrier synchronization. Thus, non-coherent FSK is universally used with fast hopping. The despread or de-hopped signal is plotted in Figure 4.5 which shows that the despread data is the same as in slow hopping. Fast hopping, although more difficult to implement, offers some advantages over slow hopping. First, unlike slow hopping, fast hopping provides frequency diversity at the symbol level which provides substantial benefit in fading channels or versus narrowband jamming. Slow hopping can obtain these same benefits through error correction coding as we will see later, but fast hopping offers this benefit before coding is applied, which can provide better performance, especially when punctured codes are used. The down side to fast hopping is that the separate integration periods collected each hop must be combined non-coherently since non-coherent demodulation is used. As a result, a loss in performance is experiences as compared to standard non-coherent FSK modulation.

The reception of FH/SS is accomplished as shown in Figure 4.3. The despread signal \( y(t) \) is obtained by multiplying the incoming signal by the hopping signal and filtering out the images:

\[
y(t) = [r(t)h(t)]_{BPF} \\
= \left[ (s(t) + n(t)) \sum_{i=-\infty}^{\infty} p(t - iT_c) \cos (2\pi f_i t + \phi_i) \right]_{BPF} \\
= s_d(t) + n'(t)
\]

where \( n'(t) \) is the noise process after despreading and filtering. 

Figure 4.5: Example of Time-Frequency Plot after Despreading
4.2 Complex Baseband Representation

As with DS/SS, the complex envelope representation can be convenient for analysis and simulation. Thus, we would like to introduce the complex baseband for FH/SS systems. Specifically, the hopping waveform can be represented in complex baseband as

$$\tilde{h}(t) = \sum_{i=-\infty}^{\infty} p(t - iT_c)e^{j(2\pi f_i t + \phi_i)}$$  \hspace{1cm} (4.4)

where \(f_i\), \(\phi_i\) and \(p(t)\) were defined earlier. For \(M\)-FSK modulation, the complex baseband version of the data signal is

$$\tilde{d}(t) = \sum_{i=-\infty}^{\infty} p(t - iT_s)e^{j(2\pi f_i t + \phi_i)}$$  \hspace{1cm} (4.5)

where \(f_i \in \{f_1, f_2, \ldots, f_M\}\) are the \(M\) symbol frequencies. The transmit signal is then

$$\tilde{s}(t) = \tilde{d}(t)\tilde{h}(t)$$  \hspace{1cm} (4.6)

and dehopping (despreading) is accomplished by \(\tilde{y}(t) = \tilde{s}(t)\tilde{h}^*(t) = \tilde{h}(t)\tilde{d}(t)\tilde{h}^*(t) = \tilde{d}(t)\) since \(|\tilde{h}(t)|^2 = 1\) assuming square pulses.

4.3 Power Spectral Density of FH/SS

The power spectral density of FH/SS can be found as

$$S(f) = S_d(f) * H(f)$$  \hspace{1cm} (4.7)
where $S_d(f)$ is the power spectral density of the data modulated carrier before hopping and $H(f)$ is the power spectral density of the hopping waveform. If we define $N$ as the number of hop frequencies it can be shown that the PSD of the hopping waveform is

$$H(f) = \frac{1}{T_c^2} \sum_{i=-\infty}^{\infty} \left| \sum_{k=1}^{N} p_k G_k \left( \frac{i}{T_c} \right) \right|^2 \delta \left( f - \frac{i}{T_c} \right) + \frac{1}{T_c} \sum_{k=1}^{N} p_k (1 - p_k) |G_k(f)|^2$$

$$- \frac{1}{T_c} \sum_{k=1}^{N} \sum_{m=1}^{N} p_k p_m \Re \{ G_k(f) G^*_m(f) \}$$

where $G_m(f)$ is the Fourier Transform of the pulsed carrier $p(t)\cos(2\pi f_m t + \phi_m)$ defined over $0 \leq t \leq T_c$ and $p_m$ is the probability of using the $m$th carrier. We can find $G_m(f)$ as (assuming $p(t)$ is a square pulse)

$$G_m(f) = \mathcal{F}\{g_m(t)\} = \mathcal{F}\{p(t)\cos(2\pi f_m t + \phi_m)\} = T_c e^{-j\pi(f - f_m)T_c} \text{sinc}((f - f_m)T_c) + T_c e^{-j\pi(f + f_m)T_c + \phi_m} \text{sinc}((f + f_m)T_c)$$

Now, if the carrier spacing is such that the spectra of $G_m(f)$ and $G_k(f)$ do not overlap for $m \neq k$ (i.e., if the hop rate $\frac{1}{T_c}$ is slow compared to the minimum carrier spacing), and we assume that all hop frequencies are equally likely we obtain

$$H(f) \approx \frac{1}{T_c^2 N^2} \sum_{i=-\infty}^{\infty} \sum_{k=1}^{N} \left| G_k \left( \frac{i}{T_c} \right) \right|^2 \delta \left( f - \frac{i}{T_c} \right) + \frac{1}{T_c} \frac{1}{N} \sum_{k=1}^{N} |G_k(f)|^2$$

Inserting for equation (4.9) for $G_m(f)$ into (4.10), the resulting PSD is

$$H(f) \approx \frac{1}{N^2} \sum_{i=-\infty}^{\infty} \sum_{k=1}^{N} \left( \text{sinc}^2 \left( i - f_k T_c \right) + \text{sinc}^2 \left( i + f_m T_c \right) \right) \delta \left( f - \frac{i}{T_c} \right) + \frac{T_c}{N} \left( 1 - \frac{1}{N} \right) \sum_{k=1}^{N} \left[ \text{sinc}^2 \left( (f - f_k)T_c \right) + \text{sinc}^2 \left( (f + f_k)T_c \right) \right]$$

Now, if we choose the frequency spacing to be an integer multiple of the hop rate for illustration purposes, we sample the sinc function at integer values
eliminating all terms except the first:

\[ H(f) \approx \frac{1}{N^2} \sum_{k=1}^{N} [\delta(f - f_k) + \delta(f + f_k)] + \frac{T_c}{N} \left(1 - \frac{1}{M}\right) \sum_{k=1}^{N} [\text{sinc}^2((f - f_k)T_c) + \text{sinc}^2((f + f_k)T_c)] \]

(4.12)

As an example, let us consider the power spectral density when BPSK with coherent frequency hopping is used. Now, from previous developments we know that the PSD of BPSK is

\[ S_d(f) = \frac{A^2 T_b}{4} \left[\text{sinc}^2((f - f_c)T_b) + \text{sinc}^2((f + f_c)T_b)\right] \]

(4.13)

In order to find the PSD of transmit signal \( S(f) \), we must convolve \( H(f) \) with \( S_d(f) \) resulting in

\[ S(f) \approx \frac{P T_b}{2N^2} \sum_{k=1}^{N} [\text{sinc}^2((f - f_k - f_c)T_b) + \text{sinc}^2((f + f_k + f_c)T_b)] + \left(1 - \frac{1}{N}\right) \frac{P T_b}{2N} \sum_{k=1}^{N} [\text{sinc}^2((f - f_k - f_c)T_b) + \text{sinc}^2((f + f_k + f_c)T_b)] \]

\[ = \frac{P T_b}{2N} \sum_{k=1}^{N} [\text{sinc}^2((f - f_k - f_c)T_b) + \text{sinc}^2((f + f_k + f_c)T_b)] \]

(4.14)

which is an intuitively satisfying result as it says that the PSD of the frequency-hopped signal is the sum of \( N \) replicas of the information signal PSD each centered at the hopping frequencies. An example is plotted in Figure 4.7 for \( R_b=1\text{Mbps} \) and hop frequencies of 11MHz, 12MHz, 13MHz, and 14MHz (i.e., 4 hop frequencies)

### 4.4 Processing Gain

As discussed in the previous chapter, processing gain is a measure which provides a short-hand description of the benefits of a particular spread spectrum format. Like with DS/SS there are multiple definitions available in the literature with subtle differences. As with DS/SS we will formally define processing gain as the ratio of the bandwidth to the bit rate

\[ PG = \frac{W}{R_b} \]

(4.15)

while we will define the number of non-overlapping frequency bands available for hopping as the bandwidth expansion factor \( N \). However, note that in most cases of interest, the two will be equivalent and the processing gain will be equal to the common definition \( PG = N \).
4.5. PERFORMANCE OF FH/SS

As with DS/SS, FH/SS results in no performance benefit in AWGN channels. This can be readily seen by examining the dehopped signal. If we use BPSK modulation, the received signal after despreading in complex baseband notation is

\[
\tilde{y}(t) = \tilde{r}(t)\tilde{h}^*(t) = \left(\tilde{h}(t)b(t) + n(t)\right)\tilde{h}^*(t)
\]

(4.16)

Now, if the hop sequence is completely random, dehopping will not impact the noise characteristics. Thus, the performance of coherent BPSK with coherent hopping results in a probability of error

\[
P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)
\]

(4.17)

and other modulation schemes also achieve the same performance as without frequency hopping. Now consider a narrowband noise jammer with bandwidth \(B\). Assuming that the jammer knows the frequency band of the signal of interest, and \(N_j = J/B\), the performance without frequency hopping is

\[
P_b = Q\left(\sqrt{\frac{1}{\frac{N_0}{2E_b} + \frac{N_j}{2E_j}}}\right)
\]

(4.18)

which is plotted in Figure 4.8 (curve labeled "Narrowband Signal") for \(\frac{E_b}{N_0} = 20\)dB and \(\frac{E_b}{N_j} = 7\)dB. We see that the presence of the jammer results in a probability of error of approximately 0.1%. Now, consider a frequency hopping signal
Figure 4.8: Performance of BPSK Narrowband Signal and BPSK Frequency-Hopped Signal in the Presence of Narrowband Interference ($E_b/N_j=7$dB, $E_b/N_o=20$dB)

which randomly hops to $N$ different frequency bands. Since the frequency hopper will land in the band of the jammer only 1 out of $N$ hops, the performance becomes

$$P_b = \frac{1}{N} Q\left(\sqrt{\frac{1}{N\frac{E_o}{2E_b} + \frac{N_j}{2E_b}}}\right) + \frac{N - 1}{N} Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$ \hspace{1cm} (4.19)

which is also plotted in Figure 4.8 for the same parameters, but allowing $N$ to vary. Obviously, as we let $N$ increase, we are impacted by the jammer less frequently, improving performance. We can also plot the performance vs. $\frac{E_b}{N_o}$ as shown in Figure 4.9. The benefit of frequency hopping is evident, although this gain is clearly diminished as $\frac{E_b}{N_o}$ increases. Note that the gains are different than DS/SS shown in the previous chapter. With DS/SS the received signal is always subject to interference albeit at reduced levels after despreading. In FH/SS the effective power level of the jammer is not reduced, rather the frequency of the jammer’s impact is reduced. This of course depends on the type of jammer being studied as we will see in Chapter 8.
4.6 Coding

Now consider the use block codes which can correct up to $e$ errors out of a block of $B$ bits. The probability of codeword error $P_w$ can be written as

$$P_w = 1 - P_c = 1 - \sum_{i=0}^{e} \binom{B}{i} (1 - P_b)^{B-i} (P_b)^i$$  \hspace{1cm} (4.20)

If frequency hopping occurs every bit (or interleaving is done across hops) errors will primarily occur when the hopping signal hops into the jammer’s band. Ideally, the error correction code can correct these errors, provided that there are not too many of them (i.e., we do not hop into the jammer’s band too often). As an example, consider the previous example but with error correction coding on top of frequency hopping. Specifically, assume $B=63$ (the code rate is $r = 36/63$), $e=5$ and $N=63$. The resulting performance of standard coherent BPSK with block coding is plotted in Figure 4.10 along with the performance of frequency hopping with coding. We can see that the combination of frequency hopping and error correction coding provides a tremendous improvement in the presence of jamming. While coding improves narrowband performance, the combination of FH and coding is especially powerful. We will examine the performance FH/SS in more detail in Chapter 8.

Figure 4.9: Performance of Narrowband BPSK Signal and Frequency-Hopped BPSK Signal in the Presence of a Narrowband Jammer ($\frac{E_b}{N_0} = 20\ dB, \ N = 64$)
4.7 Other Techniques

In addition to DS/SS and FH/SS there are several other forms of spread spectrum. Three specific forms of spread spectrum that we will discuss in this section include

- hybrid DS/FH/SS
- chirp modulation
- time hopping

4.7.1 Hybrid DS/FH/SS

Another common form of spread spectrum is known as Hybrid DS/FH/SS which combines frequency hopping and direct sequence techniques. The technique is described in Figures 4.11 and 4.12. As described in Figure 4.11 the data signal is first spread using a direct sequence technique and then further spread by hopping the center frequency to one of \( N \) hop frequencies. This technique is useful for obtaining extremely high spreading factors since it can spread the bandwidth more than either DS/SS or FH/SS alone. Additionally, this technique increases the complexity needed for an interceptor and provides benefits of both DS and FH.
4.7. OTHER TECHNIQUES

4.7.2 Chirp Modulation

Chirp modulation is another form of spread spectrum where the symbol or pulse is a continuous wave signal whose frequency increases linearly over the symbol duration. That is the frequency is

\[ f_0 + \mu t \quad 0 \leq t \leq T_s \]  

(4.21)

where \( f_0 \) is the initial frequency and \( \mu = \frac{df}{dt} \) is the rate of the chirp. The transmit signal is then

\[ s(t) = a(t)\cos\left(2\pi f_1 t + \pi \mu t^2 + \phi\right) \quad 0 \leq t \leq T_s \]  

(4.22)

where \( a(t) \) is an amplitude weighting function. When \( \mu \) is greater than zero, we term the pulse an upchirp and when \( \mu < 0 \) we term the pulse a downchirp. An example upchirp signal where \( f_0 = 10kHz \), \( \mu = 90MHz \), and \( T_s = 1ms \) is plotted (for the first 0.3ms) in Figure 4.13. Note that plot assumes a rectangular amplitude weighting function, \( a(t) = \text{rect}\left(\frac{t-T_s}{T_s}\right) \). The entire pulse is plotted in Figure 4.14 for both an upchirp (right) and downchirp (left).

To understand the benefits of chirp pulses, we must first examine the autocorrelation function (i.e., matched filter output) and the spectrum. First, let us consider the spectral properties of a chirp signal. The bandwidth of a chirp signal can be well approximated (depending on the exact nature of the weighting function and the bandwidth definition of interest) by the change in instantaneous frequency of the chirp. That is

\[ W = \mu T_s \]  

(4.23)

Assuming that \( \mu >> R_s \), the bandwidth is larger than the data rate. For example, in the previous example \( \mu = 90MHz \) and \( T_s = 1ms \). The nominal narrowband data rate is approximately \( R_s = 1kHz \). However, the chirp bandwidth is \( W = 90kHz \), nearly two orders of magnitude greater. Thus, we are clearly
increasing the bandwidth well beyond the necessary bandwidth. The processing gain (assuming one bit per symbol or $R_b = R_s$) is

$$PG = \frac{W}{R_b} = \frac{\mu}{R_b^2} \quad (4.24)$$

Another way of understanding the processing gain of the chirp signal is to examine the autocorrelation function of the chirp signal. The normalized autocorrelation of an upchirp signal (assuming a rectangular weighting function) can be shown to be [38]

$$R_s(\tau) = \frac{\sin \left( \frac{\pi B \tau (1 - |\tau|/T_s)}{\pi B T_s} \right) \cos (2\pi f_0 \tau)}{\pi B T_s} \quad (4.25)$$

An example plot (using the same parameters given earlier) is shown in Figure 4.15. From the figure we can see that the main lobe of the autocorrelation function is approximately 0.02ms or roughly $2/W$. This is a general property of chirp signals. While the duration of a symbol is $T_s$, the autocorrelation function is much more narrow and has a width of approximately $2/W$. This property is termed time compression and makes chirp signals useful for radar, resistant to interference and multipath. The normalized autocorrelation function is plotted in Figure 4.16 for various time-bandwidth products, e.g., $WT_s = 10, WT_s = 20$, and $WT_s = 100$. It is clear that increasing the time-bandwidth product narrows the autocorrelation function at a rate equivalent to the bandwidth increase.

Returning to the spectral properties of the chirp signal for a moment, from the plot in Figure 4.15 we can see that the autocorrelation function resembles the
sinc function. As a result, the power spectral density will approach a rectangular pulse in frequency. However, the finite duration of the pulse will cause the signal to have sidelobes. This can be seen in Figure 4.17 where we plot the magnitude spectrum for both upchirp and downchirp signals using the same parameters as used previously. From the plots we can see that (a) the spectra for the up and down chirps are essentially identical; (b) the bandwidth of the main lobe of the spectrum has a width of approximately 90kHz ($\mu T_s$); and (c) the pulse shape $a(t)$ introduces large sidelobes. These sidelobes can be reduced by applying a weighting function (i.e., window) to the chirp. Alternatively, increasing the time-bandwidth product also reduces the sidelobes. For example, an upchirp with $f_1 = 10kHz$, $\mu = 1MHz$, and $T_s = 90ms$ provides the same bandwidth, but a much higher time-bandwidth product (processing gain) of $BT = PG = 8100$. The spectrum is plotted in Figure 4.18. Comparing it to Figure 4.17 we can see that the basic shape is the same, but the sidelobes are significantly reduced due to the increased pulse duration.

### 4.7.3 Time Hopping (Ultra Wideband)

The final spread spectrum technique that we will discuss is Time-Hopped Spread Spectrum (TH/SS). Time hopping is typically used with pulse position modulation (PPM), thus we briefly describe PPM first. Binary PPM is described in Figure 4.19. With PPM, the position of the pulse is moved forward or backward from the nominal symbol time by $\delta$ seconds based on the data being sent. Provided that the pulses are non-overlapping, the modulation scheme is orthogonal.
and achieves the performance equivalent to any orthogonal modulation scheme. In other words

\[ P_b = Q \left( \sqrt{\frac{E_b}{N_0}} \right) \]  

(4.26)

However, depending on the pulse shape used, slightly better performance can be obtained with non-orthogonal pulses. That is, with a proper choice of \( \delta \) the correlation can be slightly negative, providing better performance than orthogonal modulation. Specifically,

\[ P_b = Q \left( \sqrt{\frac{E_b}{N_0}} (1 - R_x(2\delta)) \right) \]  

(4.27)

where \( R_x(\tau) \) is the normalized autocorrelation function of the pulse shape used and \( \delta \) is the time shift for the two symbols. Figure 4.20 plots \( R_x(\tau) \) for the Gaussian pulse. We can see that the largest negative value is approximately \(-0.74\) which occurs at a delay of 0.235ns. Thus, we should set \( \delta = 0.1625 \) for best performance.

To improve the probability of intercept, as well as to reduce the impact of jamming, time hopping can be added to PPM. In order to allow additional time modulation for time hopping, a frame time is defined as shown in Figure 4.21. A single pulse is transmitted each frame where the frame duration \( T_f \) is much larger than the pulse duration \( T_p \). The bandwidth is determined by the pulse duration \( T_p \) while the data rate is proportional to the pulse repetition rate \( R_f = 1/T_f \). Time hopping modulates the position of the pulse within the frame as shown in Figures 4.21. Further, this additional modulation is pseudorandom which makes unauthorized detection and jamming more difficult.

The frame size depends on the number of possible time hopping positions which have granularity \( T_c \) (Figure 4.21). Additionally, multiple \((N_s)\) pulses can be transmitted per information bit, which decreases the data rate, increasing the spreading factor \( B/R_0 \). This is described in Figure 4.19. The final form of
4.7. OTHER TECHNIQUES

Figure 4.15: Autocorrelation of Upchirp Signal ($f_o = 10kHz$, $\mu = 90 MHz$, $T_s = 1ms$)

the transmit signal can be expressed as

$$s(t) = \sum_{i=-\infty}^{\infty} A_p \left( t - iT_f - c_i T_c - \delta d_{i/N_s} \right)$$

(4.28)

where $c_j$ represents the pseudorandom time hopping code and $d_j$ represents the data modulation. Demodulation is accomplished by first de-hopping the signal and then performing maximum likelihood detection on the PPM signal.

The processing gain can be approximated as the uncoded bandwidth expansion factor which is equal to the product of the number of time slots in the time-hopping code $N_h = T_f/T_p$ and the number of frames (pulses) per bit $N_s$:

$$N = \frac{R_f}{R_b} = N_h N_s$$

(4.29)

To see this, consider the performance (i.e., the probability of bit error) of TH/SS in the presence of tone interference [39]:

$$P_b = Q \left( \sqrt{\frac{2E_b}{N_f}} \right)$$

(4.30)
where \( E_b = P T_b \) is the received energy per bit, \( N_j = J/W \) is the interference power spectral density and \( W \) is the signal bandwidth. The signal bandwidth is determined by the pulse shape and can be approximated by the inverse pulse duration:

\[
B = \frac{1}{T_p} \tag{4.31}
\]

The bit rate is related to the frame rate \((1/T_F)\) and the number of frames (pulses) per bit \( N_s \):

\[
R_b = \frac{1}{T_F N_s} \tag{4.32}
\]

Using these definitions in (4.30), we have

\[
P_b = Q \left\{ \sqrt{\frac{2 P T_F N_s}{J T_p}} \right\} \tag{4.33}
\]

Using the definition \( N_h = T_F/T_p \) the number of time-hop slots and the signal-to-interference ratio \( SIR = P/J \), we finally have

\[
P_b = Q \left\{ \sqrt{2 SIR \cdot N_h N_s} \right\} \tag{4.34}
\]
4.8. **Conclusions**

To this point we have discussed two primary versions of spread spectrum, namely direct sequence spread spectrum (Chapter 3) and frequency-hopped spread spectrum. In this chapter we briefly introduced frequency-hopped spread spectrum and other less common forms of spread spectrum. We will discuss the performance of FH/SS in detail in Chapter 8. However, before we do that, we will discuss spreading waveforms, synchronization and tracking, three very important aspects of spread spectrum. Specifically, in the next chapter we will discuss the properties of the spreading waveforms that are vital to the proper operation of spread spectrum as well as techniques for generating spreading waveforms.

Figure 4.17: Example Spectra for Upchirp (left) and Downchirp (right) Waveforms ($f_0 = 10kHz$, $\mu = 90MHz$, $T_s = 1ms$, $BT = 90$)

Clearly as we increase the bandwidth expansion factor, either by increasing the number of frames per bit or the hop slots per frame, improves performance for a fixed SIR.

4.8 **Conclusions**
Figure 4.18: Example Spectrum for Upchirp Waveform with $BT = 8100$

Figure 4.19: Pulse Position Modulation
Figure 4.20: Autocorrelation Function of Example Gaussian Pulse
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Figure 4.21: Illustration of Spectrum Spreading through Time Hopping
Bibliography


